Field Theory of Matter. II

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A qualitative dynamical interpretation is given for observed regularities of nonleptonic phenomena in strong, electromagnetic, and weak interactions.

INTRODUCTION

THE strongly interacting particles exhibit various approximate regularities that appear to have rather special dynamical origins. Nonleptonic decays are governed by the $\Delta T=\frac{1}{2}$ rule but with appreciable $\Delta T=\frac{3}{2}$ admixture, which is not naturally explained as a symmetry property of the weak interactions. Electromagnetic mass differences show a dominant $\Delta T=1$ effect in the approximately equal spacing of Σ^-, Σ^0, and Σ^+, but $\Delta T=2$ must also be significant to account for the very nature of the pion mass spectrum ($\pi^+=\pi^-\neq\pi^0$). The baryon ($\frac{1}{2}^+$) and baryon resonance ($\frac{3}{2}^+$) multiplets are approximately equally spaced when they are regarded as U or V multiplets, in the sense of the three isotopic spins T, U, V that characterize SU_3 symmetry. Thus, equal spacing for the $U=1$ baryon multiplet N^0, $\frac{1}{2}(3^{1/2}\Lambda+\Sigma^0)$, Ξ^0 or the $V=1$ multiplet Ξ^-, $\frac{1}{2}(3^{1/2}\Lambda-\Sigma^0)$, N^+ expresses the Gell-Mann–Okubo (GO) mass relation. It is a directly observable property of, say, the $U=\frac{3}{2}$ resonance multiplet N^{*-}, Y^{*-}, Ξ^{*-}, Ω^-. This means that the mechanism predominantly responsible for the mass splittings obeys $\Delta U=1$ and $\Delta V=1$.

We shall give a qualitative dynamical interpretation of these regularities in the framework of a recently proposed field theory of matter.[1] In doing this we face another such question, but one that, for the moment, lacks a definite experimental basis. How does the breakdown of the underlying W_3 symmetry produce predominantly SU_3 symmetry?

BROKEN SYMMETRIES

W_3 and SU_3

The first in the hierarchy of interactions couples the gauge field B to the nucleonic charge-bearing fields $\psi_a(N=\pm1)$ and $V_a(N=\pm2)$, $a=1, 2, 3$. That interaction is invariant with respect to the two independent internal symmetry groups $U_3(\psi)$ and $U_3(V)$. Such is the underlying symmetry W_3. It is broken by the interaction that exchanges nucleonic charge between ψ and V in accordance with the unitary structure $\bar{\psi}_3(\bar{\psi}V)$ and its adjoint. Although this is a strong interaction, it will be expedient to discuss its implications in the language of perturbation theory.

The low-lying states that one identifies with degenerate families of physical particles are created by combinations of fields with opposite signs of nucleonic charge, as in $\bar{\psi}_a\psi_b$, $\bar{\psi}_aV_b$, and $\bar{\psi}_a\bar{\psi}_b\psi_cV_d$. All these combinations are characterized by invariance under common phase transformations of ψ and V,

$$\psi_a\to e^{i\alpha}\psi_a,\quad V_a\to e^{i\alpha}V_a,\quad a=1,2,3\,,$$

as distinguished from the phase transformation of nucleonic charge. Thus the total nucleonic charges associated with the ψ and V fields in these states are related by

$$\tfrac{1}{2}N(V)=-N(\psi)=N\,.$$

If we assume that all other types of states (such as the triplets generated by ψ_a and V_a) are much more massive, the most important effect on the low-lying states, of the W_3 destroying interaction, comes from the part that is invariant under the common phase transformation. This is represented by the set of perturbation terms symbolized as

$$(\bar{\psi}_3\psi_3)^n[(\bar{\psi}V)(\bar{V}\psi)]^n\quad n=1,\cdots.$$

The product of $2n$ field operators at distinct space-time points that is implied by $(\bar{\psi}_3\psi_3)^n$ can be decomposed into irreducible parts by means of the vacuum expectation value. Thus, $(\bar{\psi}_3\psi_3)^n$ is additively represented by a number $\langle(\bar{\psi}_3\psi_3)^n\rangle$; irreducible operator pairs

$$[\bar{\psi}_3\psi_3]=\bar{\psi}_3\psi_3-\langle\bar{\psi}_3\psi_3\rangle,$$

multiplied by numbers which are the vacuum expectation values $\langle(\bar{\psi}_3\psi_3)^{n-1}\rangle$; and so forth. When all terms in the perturbation series are rearranged in this way, the resulting functional form is

$$f_0((\bar{\psi}V)(\bar{V}\psi))+[\bar{\psi}_3\psi_3]f_1((\bar{\psi}V)(\bar{V}\psi))+\cdots.\quad(1)$$

A vacuum expectation value becomes singular as two points coincide. If the symmetry destroying interaction is sufficiently localized in space and time, owing to the dominance of very massive states, the consecutive terms in the series (1), which involve successively fewer replacements of operator products by vacuum expectation values, may be of diminishing numerical importance. Under such circumstances, the primary effect of the mechanism that breaks W_3 symmetry is to introduce a dynamical regime of SU_3 symmetry, as the symmetry

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[1] Julian Schwinger, Phys. Rev. Letters **12**, 237 (1964) and Phys. Rev. **135**, B816 (1964). See also *Symmetry Principles at High Energy* (W. H. Freeman and Company, Inc., San Francisco, 1964).

group that governs common[2] transformations of ψ_a and V_a in the coupling term $f_0((\bar\psi V)(\bar V\psi))$.

The same mechanism has the secondary effect of breaking down SU_3 symmetry. This aspect of the perturbation is dominated by the term with $[\bar\psi_3\psi_3]$ as a factor. The latter responds as a component of a three-dimensional Euclidean vector under the SU_2 subgroups that govern transformations in the 23 plane (U isotopic spin) and the 31 plane (V isotopic spin). Accordingly, matrix elements of this perturbation will obey just those selection rules, $\Delta U=1$, $\Delta V=1$, that are observed in the dominant part of the mass splitting that violates SU_3 symmetry. A relative measure of the higher perturbation terms can be had by comparing the 8 MeV excess in the mass of Λ, over the value given by the GO formula, with the average of the mass splittings between Σ and N, Ξ and Σ. This ratio is 1:23. If a similar ratio connects the first two terms of (1), the intervals between the SU_3 multiplets of a common W_3 representation should be several BeV. The W_3 concept will acquire an observational basis when one finds some evidence of this larger pattern among the particles.

The dynamical processes that result in broken SU_3 symmetry possess the residual symmetry that is described by the invariance group U_2, generated by the isotopic spin T (SU_2) and the hypercharge Y.

Electromagnetism and SU_2

The electric current vector is formed additively from the operator products $\bar\psi_1\psi_1$ and $\bar V_1 V_1$ (omitting spinor and vector indices). A part of the current is proportional to the electromagnetic vector potential A. Electromagnetic mass displacements and the breakdown of SU_2 symmetry are produced by electromagnetic vacuum fluctuations through this induced current, together with the second order effect of the linearly coupled potential. The nature of the two mechanisms is symbolized by

$$(\bar\psi_1\psi_1+\bar V_1V_1)+(\bar\psi_1\psi_1+\bar V_1V_1)^2. \tag{2}$$

The operators $\bar\psi_1\psi_1$ and $\bar V_1V_1$ behave as components of three-dimensional Euclidean vectors with respect to T isotopic spin transformations (12 plane). Therefore they have matrix elements obeying $\Delta T=1$. The products of these operators also possess matrix elements with $\Delta T=2$. But such operator products can be decomposed into irreducible parts through the introduction of the vacuum expectation value. A term such as $[\bar\psi_1\psi_1]\langle\bar\psi_1\psi_1\rangle$ is characterized by $\Delta T=1$. These vacuum expectation value contributions, and the $\Delta T=1$ effect, will dominate to the extent that the electromagnetic mechanism is localized. Thus, the long-range nature of electromagnetic action would seem to explain the comparative importance of $\Delta T=2$ effects, as measured by the 1:3 (mass)^2 ratio of $\pi^+-\pi^0$ and K^0-K^+.

Weak Interactions and Hypercharge

The weak interactions of the strongly interacting particles are described by a coupling of the charged vector field Z with currents of the form indicated by $\bar\psi_1\psi_2+\bar V_1V_2(\Delta T=1,\ \Delta Y=0)$ and $\bar\psi_1\psi_3+\bar V_1V_3$ ($\Delta T=\frac{1}{2}$, $|\Delta Y|=1$). The self-action of these currents through the intermediary of the Z field contains a part symbolized by

$$(\bar\psi_2\psi_1+\bar V_2V_1)(\bar\psi_1\psi_3+\bar V_1V_3)$$

and its adjoint. This perturbation destroys the conservation of hypercharge ($|\Delta Y|=1$) and induces isotopic spin transitions with $\Delta T=\frac{3}{2}$ and $\frac{1}{2}$. The decomposition of the operator by means of vacuum expectation values gives the term

$$\bar\psi_2\langle\psi_1\bar\psi_1\rangle\psi_3+\bar V_2\langle V_1\bar V_1\rangle V_3 \tag{3}$$

and its adjoint, which is characterized by $|\Delta Y|=1$ and $\Delta T=\frac{1}{2}$. We conclude from the observed dominance of the latter effect that the current self-coupling is effectively localized, or that the Z-field excitations are quite massive. The relative importance of the $\Delta T=\frac{3}{2}$ effect is measured by the amplitude ratio for $K^+\to\pi^++\pi^0$ and $K_1{}^0\to\pi^++\pi^-$, which is 1:23.

We recall the detailed structure of the fermion current that is coupled to $\bar Z_\mu$,

$$\bar\psi_1\gamma^\mu(1+i\gamma_5)\psi_2+\bar\psi_1\gamma^\mu(1-i\gamma_5)\psi_3.$$

The opposite signs of $i\gamma_5$ in the two terms should be noted. The $\Delta T=\frac{1}{2}$ contribution derived from the second-order effect of this coupling contains the Green's function of the Z field and of the ψ_1 field. When the product of the Green's functions is approximated by a four-dimensional delta function, the resulting localized interaction is proportional to

$$-(\tfrac{1}{8})\bar\psi_2\gamma^\mu(1+i\gamma_5)\gamma_\mu(1-i\gamma_5)\psi_2$$

with its adjoint, which equals

$$\bar\psi_2(1-i\gamma_5)\psi_3+\bar\psi_3(1+i\gamma_5)\psi_2. \tag{4}$$

The complete $\Delta T=\frac{1}{2}$, CP invariant perturbation is obtained by adding the parity-preserving contribution of the V field, which is represented by

$$\bar V_2{}^\mu V_{\mu 3}+\bar V_3{}^\mu V_{\mu 2}. \tag{5}$$

No general statement can be made about the relative magnitude of the parity-violating and parity-preserving effects, particularly since the strong interactions of the broken SU_3 symmetry scheme are of decisive influence on the observed phenomena.[3]

[2] The additional possibility of independent phase transformations for ψ and V is described by the nucleonic charge phase transformation and by the common phase transformtaion that leaves invariant the low-lying states.

[3] Indeed, the existence of these processes seems to involve the breakdown of SU_3 symmetry, beyond the mere role of supplying the necessary energy. Thus the weak parity-violating coupling between K^* and π that accounts for $K_1{}^0\to\pi+\pi$ (see Ref. 4)

PHENOMENOLOGY

In view of the serious technical obstacles to performing field theory calculations from first principles, it is useful to convert the characteristic ideas of this field theory of matter into a corresponding phenomenology. Such a program is suggested by the structure of the electromagnetic interaction. The electromagnetic potential A_μ is coupled to the electric current vector

$$j^\mu = e[-\bar{\psi}_1\gamma^\mu\psi_1 - i(\bar{V}_1^{\mu\nu}V_{1\nu} - \bar{V}_{1\nu}V_1^{\mu\nu})].$$

This operator generates meson states of spin-parity 1^-, as represented by phenomenological fields U_{11}^μ. Thus, one could attempt to describe all linear electromagnetic interactions as proceeding through the fields of 1^- mesons with suitable quantum numbers. The least massive of these, ω and ρ^0, might be of major importance in this description. Similarly, the vector and pseudovector currents that are coupled to the Z field can be represented approximately by the phenomenological fields of known 1^- and 0^- mesons. These couplings would describe all such weak interactions.[4] The so-called Goldberger-Treiman relations are an immediate consequence of this point of view which is, in a sense, a return to the original idea of Yukawa.

The nonleptonic weak interaction (3), as clarified by (4) and (5), can be represented by suitable components of the phenomenological fields associated with 0^- and 0^+ mesons. Particles of the latter type have not been identified with certainty,[5] but these excitations will exist somewhere in the mass spectrum. Such scalar fields will also represent the part of the perturbation (2) that produces $\Delta T=1$ electromagnetic mass displacements, as well as the portion of (1) that generates the mass splittings of broken SU_3 symmetry.[6] Scalar fields can also be used to describe the break-down of W_3 symmetry.

This is an outline of a phenomenological field theory, which gives quantitative expression to the general ideas that have been expressed here. It will be developed in another publication.

would vanish if the K and π masses were equal, owing to the transversality of the four-vector K^* field. A similar remark applies to baryon decays. As we shall discuss elsewhere, simple models of parity-conserving decays depend upon coupling constant differences that would be zero were SU_3 symmetry not broken. [*Added in proof*. See J. Schwinger, Phys. Rev. Letters **13**, 355 and 500 (1964).]

[4] This idea is used to compute the absolute rate for the $\Delta T=\frac{3}{2}$ process $K^+\to\pi^++\pi^0$ in J. Schwinger, Phys. Rev. Letters **12**, 630 (1964).

[5] Some possibilities are discussed by S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

[6] A perturbation that is a numerical multiple of a phenomenological boson field generates a displacement of that field, which is its nonvanishing vacuum expectation value. We propose the word *vacuon* for this property. (Despite its hybrid etymology, such a terminology seems preferable to the use of a biologicographic argot.) The vacuon concept is not new [J. Schwinger, Phys. Rev. **104**, 1164 (1956); Ann. Phys. (N. Y.) **2**, 407 (1957); A. Salam and J. Ward, Phys. Rev. Letters **5**, 390 (1960)], but its most effective application is that of Ref. 5. We have now connected thie phenomenological procedure with a fundamental dynamical theory.

Notes added in proof. (1) Some statements about the group SU_3 need clarification and correction. For each field, ψ and V, the generators of SU_3 transformations are obtained from the U_3 generators by forming a traceless combination, with the aid of the corresponding nucleonic charge,

$$\psi:\ {}'T_{ab}=T_{ab}-\tfrac{1}{3}\delta_{ab}(-N)$$

$$V:\ {}'T_{ab}=T_{ab}-\tfrac{1}{3}\delta_{ab}(-\tfrac{1}{2}N).$$

The complete generators

$$T_{ab}=T_{ab}(\psi)+T_{ab}(V)$$

are related by

$${}'T_{ab}=T_{ab}-\tfrac{1}{3}\delta_{ab}F,$$

where

$$F=-N(\psi)-\tfrac{1}{2}N(V)=T_{11}+T_{22}+T_{33}.$$

We shall call this quantity field charge. Unit field charge is assigned to $\bar{\psi}$ and $\bar{V}$,

$$[\bar{\psi},F]=\bar{\psi},\quad [\bar{V},F]=\bar{V},$$

while ψ and V carry the opposite sign of charge. Hypercharge and electrical charge are identified as

$$Y=T_{11}+T_{22},\quad Q=T_{11},$$

and

$$Q-\tfrac{1}{2}Y=\tfrac{1}{2}(T_{11}-T_{22})=T_3.$$

The charges F, Y, and Q have integer eigenvalues. If an SU_3 hypercharge ${}'Y$ is defined by

$${}'Y={}'T_{11}+{}'T_{22}=Y-\tfrac{2}{3}F,$$

we can give the electrical charge the alternative forms

$$Q=T_3+\tfrac{1}{2}Y=T_3+\tfrac{1}{2}{}'Y+\tfrac{1}{3}F.$$

[Field charge is evidently related to what has been called triality. See, for example, G. Baird and L. Biedenharn, *Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, 1964). Triality is only conserved modulo 3, however.]

The characteristic property of low-lying states that is noted in the text can now be restated as: Low-lying states carry zero field charge. For such states there is no distinction between ${}'T_{ab}$ and T_{ab}. In particular, ${}'Y=Y$. The U_3 matrices t_{ab} and the SU_3 matrices ${}'t_{ab}$ that are associated with the fields differ, however. [I am indebted to A. Radkowski for a comment on this point.] Thus,

$$[\bar{\psi},T_{ab}]=t_{ab}\bar{\psi},\quad [\bar{\psi},{}'T_{ab}]={}'t_{ab}\bar{\psi},$$

where

$$(t_{ab})_{cd}=\delta_{ca}\delta_{bd}$$

and

$${}'t_{ab}=t_{ab}-\tfrac{1}{3}\delta_{ab}.$$

The electric charge and hypercharge matrices are

$$q=t_{11}=\begin{pmatrix}1&&\\&0&\\&&0\end{pmatrix},\quad y=t_{11}+t_{22}=\begin{pmatrix}1&&\\&1&\\&&0\end{pmatrix}$$

and not

$$'t_{11}=q-\tfrac{1}{3},\quad 't_{11}+'t_{22}=y-\tfrac{2}{3}.$$

The field charge F is conserved, in addition to the operators $'T_{ab}$, in the dynamical regime that has been described as governed by SU_3 symmetry. Accordingly, the relevant symmetry group is U_3, rather than SU_3. Stable particles with unit field charge will exist in this idealization. In particular, the operators ψ_a and V_a generate triplets of fermions and bosons, respectively. When the full dynamical effect of the ψ–V coupling is included, however, F ceases to be conserved and the triplets become highly unstable, particularly if they are quite massive. Examples of possible decay modes are (field symbols designate triplet particles)

$$\psi_{1,2}\rightarrow\Lambda+\bar{K},\quad \psi_3\rightarrow\Lambda+\eta$$

and

$$V_{1,2}\rightarrow\Lambda+\Xi,\quad V_3\rightarrow\Lambda+\Lambda.$$

Such resonant states may be observed eventually. It is the negative aspect of these predictions that needs immediate emphasis: Stable or long-lived triplets should not exist. [Compare T. D. Lee (to be published).]

(2) It is asserted in the text that the various symmetry-destroying mechanisms can be represented by suitable vacuons. While that description conveys the major effect of these perturbations one should not overlook its incompleteness. Thus, the operator $\bar{\psi}_3\psi_3 f_1((\bar{\psi}V)(\bar{V}\psi))$ is only approximately represented by a numerical multiple of $\bar{\psi}_3\psi_3$ or the corresponding scalar field. The incompleteness of the vacuon description is particularly significant for parity-conserving weak decays, since a simple vacuon treatment gives no effect. This is a consequence of detailed cancellations, which express the possibility of absorbing the weak scalar vacuon into the strong vacuon responsible for broken SU_3 symmetry. It is easy to underestimate the extent of the cancellation and just this happened in the recent notes that are cited in Ref. 3. Parity-conserving decays were there ascribed primarily to the breakdown of SU_3 coupling constant relations. But, when the relevant mechanism is represented by the vacuon that dominates the mass splittings of broken SU_3 symmetry, there is an additional weak contribution that cancels this source of parity conserving decays. Something similar occurs in the vacuon treatment of singlet-octuplet mixing. It is necessary to base the dynamical explanation of parity-conserving nonleptonic decays on the fact that the symmetry-destroying mechanisms implied by the theory cannot be represented precisely by the numerical displacement of scalar fields.